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On the Impact of Philosophical Conceptions on Mathematical Research: The Case of Condillac and Babbage*

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Abstract

The possible impact of general philosophical ideas on the choice of research subjects in mathematics is the topic of this paper. I examine a specific case in which the philosophical background is provided by a discussion on the role of language in science, which is associated with the work of Condillac. The area of mathematics considered is functional equations, a difficult chapter of mathematical analysis that began to be developed between the end of the eighteenth and the beginning of the nineteenth century. The researcher is Charles Babbage (1791-1871), a young British mathematician deeply interested in the recent work of French mathematicians and philosophers of science.

In a series of papers published in 1815-1816 Babbage developed a 'language' specifically designed for the treatment of functional equations. However, the dynamic debate on the relationship between language, science and the possibilities of a theory of signs, along the first two decades of the nineteenth century, as well as shortcomings in Babbage's approach, conspired against his interesting research approach progressing further. Finally, on the basis of the previous discussion, I make some historical remarks on the transmission of scientific information between France and Britain in these two interesting decades.

Keywords: philosophy - mathematics - Condillac - Babbage

Resumen

El tema de este trabajo es el posible impacto de grandes concepciones filosóficas sobre la elección de tópicos de investigación en el campo de la matemática. Específicamente, me ocupo de un caso en el que la discusión de las ideas de Condillac acerca del rol del lenguaje en la ciencia proporciona el trasfondo filosófico. El período histórico se sitúa entre fines del siglo dieciocho y principios del diecinueve y el área de investigación matemática es la teoría de las ecuaciones funcionales, entonces un capítulo difícil y nuevo del análisis matemático. El investigador considerado es Charles Babbage (1791-1871), un joven matemático inglés profundamente interesado en el trabajo contemporáneo de los matemáticos y filósofos de la ciencia franceses.

En una serie de trabajos publicados entre 1815 y 1816 Babbage desarrolló un 'lenguaje' específicamente diseñado para la resolución de ecuaciones funcionales. Sin embargo, el dinámico debate de las dos primeras décadas del siglo diecinueve sobre las relaciones entre el lenguaje, la ciencia y una posible teoría de los signos, conjuntamente con limitaciones técnicas percibidas en el enfoque de Babbage, conspiraron contra el desarrollo de la interesan-

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te línea de investigación que él había propuesto. Finalmente, en base a la discusión anterior, hago algunas observaciones de carácter histórico sobre el proceso de transmisión de la información científica entre Francia e Inglaterra en esas dos interesantes décadas.

Palabras clave: filosofía - matemáticas - Condillac - Babbage

1. Introduction

Contrary to the case of algebraic equations, where the unknown is a number, in functional equations the unknown entity is a function. For example,

$$\varphi(x.y) = \varphi(x) + \varphi(y),$$

where x and y are numbers, is a functional equation the solution φ of which expresses the simple property of mapping products of numbers into sums involving these individual numbers. As it is well known, the logarithmic function has such property:

$$\ln(x.y) = \ln(x) + \ln(y),$$

therefore,

$$\varphi(t) = \ln(t),$$

is a solution of that functional equation; t is a numerical variable.

In functional equations of a more general form the unknown function φ may depend on one or several variables through specified functions; it may contain in its expression integrals or derivatives of different orders, integral or fractional, of φ with respect to each of its variables and each term of the functional equation may also be multiplied by known functions of one or several variables. Clearly, a linear differential equation, that is, an equation in which a weighted sum of the unknown function φ and some of its derivatives is equated to a given function, is a very special case of a functional equation. There are specific methods to deal with differential equations but, generally, they are not applicable to the more general class of functional equations.

Mathematical problems that can be formulated in terms of functional equations appeared already in the late antiquity; Pappus of Alexandria, for example, proposed one such problem. However, to be formulated in an analytic form, these problems had to wait until a far more recent date, when the tools of mathematical analysis were sufficiently developed to allow for a meaningful representation of them in terms of functional equations.

As methods of mathematical physics began to be developed in the late eighteenth century, D'Alembert, Laplace, Monge and other prominent mathematicians began to confront problems leading to functional equations. Through their work they realized that these equations, even some with a very simple form, can be extremely difficult to solve. Several mathematicians, particularly Laplace, attempted to find general ways of solving functional equations by reducing them to alternative mathematical forms for which general methods of

solution had already been developed. It was soon realized that such reduction led to problems that were often even more difficult to handle than the initial ones, suggesting that, perhaps, functional equations may have serious intrinsic difficulties.

For this reason, some mathematicians began to explore the possibility of creating new, possibly more general approaches, for the solution of these equations. One of them was Charles Babbage, who around the middle of the 1810s was a young and gifted student of mathematics at the University of Cambridge. In parallel with his research endeavours he, together with some of his friends, made serious efforts to help updating English traditional views on mathematical analysis through the introduction of some new conceptions and advances recently made in France.

In this paper I argue that in his research on functional equations, which Babbage published in two long and difficult papers in 1815-1816, he operated from within the circle of ideas on the role of language in science proposed by Condillac. These ideas, developed since around 1780, had been discussed in several of Condillac's works, particularly in *La Logique* and, with even more relevance to mathematics, in his important work *La Langue des calculs*, which became available only in a posthumous edition of 1798.

As I explain with some more precision later, Condillac postulated that every science is no more than a 'language' with specific rules. This conception of science was adopted—implicitly or explicitly—by several leading scientists attempting to reformulate important areas of research. Among them, were Linnaeus, in botany; Lavoisier, in his development of a language specifically adapted to describe chemical reactions (Crosland 2006), and Lanz and Betancourt, in their original essay on an abstract theory of the composition of mechanical machines (Ortiz 1999). The idea of a 'language', the rules of operation of which apply to some 'atomic' elements defined in its realm, attracted also the attention of Monge. By the end of the eighteenth century he used these ideas in his formulation of descriptive geometry, a new branch of geometry dealing with the representation of three dimensional objects¹ on a two dimensional support; for example, on a sheet of paper. From there, their shape could be easily 'read' by anyone, anywhere, knowing just the set of representation-rules formulated by Monge. We could say, by understanding Monge's 'language'.

I argue that these successful experiences are at the heart of Babbage's attempt to formulate a 'language' specifically designed for the determination of a function \emptyset implicitly defined by a given functional equation. With this objective in mind, he defined a complex, powerful and often effective 'language' which he used to solve difficult functional equations, most of which had never been solved before. Nevertheless, there were instances in which, inexplicably, his machinery broke down. Unlike Lavoisier, Babbage had not—and could not

¹ In applications these objects were, mainly, pieces of machinery or solids relevant to civil engineering constructions.

in the 1810s—sufficiently clarify the basic problems underlying his difficult subject, and hence the new language he was proposing.

However, in the 1810s Babbage began developing his language-oriented method in mathematics and a serious question began to be asked in France on Condillac's recipes. De Gérando, and later more brilliantly Destutt de Tracy, the new leader of the French *Idéologues*, proposed a far richer and complex blueprint for scientific research. I also argue in this paper that Babbage met with those new and more powerful conceptions towards 1820, after writing his 1815-1816 paper, and that the criticisms levelled against Condillac's ideas affected deeply the new linguistic-oriented, or algebraic, line of research he had attempted to introduce in pure mathematics in 1815.

Finally, I remark that Babbage's complex research path, developed in the immediately post-Napoleonic period, suggests that communications between France and Britain—at least on current developments in mathematics and the philosophy of mathematics—may not have been as wide and sound as some authors (de Beer 1960) have suggested in the past.

2. Language and algebra in Condillac's circle of ideas

In his unfinished book *La langue des calculs*, published posthumously in 1798, Condillac set himself the task of finding the *grammar of algebra*. This was, indeed, a most difficult task, which may explain why the book remained unfinished. In the work Condillac declared that his studies on mathematics were really not an objective in themselves. Rather, they were a step towards fulfilling a much larger goal: to show how all sciences could be given the accuracy often assumed to be exclusive to mathematics (Condillac 1798, p. 8). In the background of his expectation—that 'metaphysical' analysis and mathematical analysis were equivalent, and consequently that one and the same analysis would apply to them (Condillac 1798, p. 218)—lies the possibility of a transcendental system of calculation with ideas, rather than merely with numbers. According to Condillac: "[c]ertainement calculer c'est raisonner, et raisonner c'est calculer: si ce sont-là deux noms, ce ne sont pas deux opérations" (Condillac 1798, pp. 226-227). He warned, however, that because of the nature of the ideas involved, analysis in metaphysics is definitely more difficult than in mathematics.

Condillac closed the first part of *La langue des calculs* stating that in it he had played the role of a *grammarian*, as algebra is nothing but a language. He stated his belief that he had achieved his goal of discussing the grammar of algebra and reflected on the fact that languages are nothing but more or less perfect analytical methods. Regarding language as an analytical method, i.e., identifying it with analytical logic, Condillac wished to give it the power enjoyed by the more elaborate model of algebra. That is why a thorough analysis of the language of algebra was, for him, a necessity. If the language associated with a given science could be taken to a higher degree of perfection, then anyone 'speaking' the language would be able to understand that particular science

with much greater precision. Therefore, for him to create a science was nothing more than creating a language, arguing that to study a science is to learn “[u]ne langue bien fait” (Condillac 1798, p. 228). In *La langue des calculs* Condillac also inscribed his well known statement: “[T]oute langue est une méthode analytique, et toute méthode analytique est une langue” (Condillac 1798, p. 1). These proclamations called for reconsideration, and some of his followers attempted to do just that.

3. Babbage’s reception of Condillac’s program

By 1810 onwards, Babbage became deeply interested in contemporaneous developments in his discipline in France and even participated in the translation into English of a standard French mathematical analysis textbook, Silvestre Lacroix’s² treatise on calculus (Lacroix 1816), which had a profound influence on the renewal of the teaching of mathematical analysis in England.

In 1815-1816, in an ‘Essay’ published in the *Philosophical Transactions* of the Royal Society (Babbage 1815-1816)³ he introduced the specific language designed to solve functional equations in one or several variables mentioned before.⁴ I claim that his results reflected both mathematical and philosophical ideas informed by his direct or indirect reading of Condillac’s work of the late 1700s.⁵ To construct his ‘language’ he formulated a tightly structured system of notation (Ortiz 2007) for which he adopted the concept of *function*, not that of quantity, as ‘atomic’ element. With this choice Babbage anticipated some basic ideas on functional analysis and the theory of abstract spaces which would be developed much later. In the exposition of his original ideas Babbage displayed a fairly sophisticated and modern conception of algebra and the approach he proposed facilitated the reduction of the solution of complex functional equations to some form of algebraic manipulation.

4. Language and signs: early fractures in Condillac’s program

However, towards the end of the 1790s conceptually deep fractures began to appear between Condillac and a new generation of his philosophical disciples, the so-called *Idéologues*. From different angles and perspectives, members of this school were attempting the construction of a ‘science of ideas’.

If an understanding of the origin of language had been the subject of a substantial literature in the late eighteenth century (Harnois 1929, Kuehner

² By the time he wrote this book Lacroix was close to the *Idéologues*.

³ Babbage’s works have been compiled by Martin Campbell-Kelly in an excellent edition, with useful index and notes, published in 1989 (Babbage 1989); the 1815-1816 papers are included in its volume one.

⁴ For the historical development of functional equations, see Dhombres (1986); Babbage’s mathematical work is reviewed in Dubbey (1978); details on the mathematics specific to Babbage’s approach to functional equations are given in Ortiz (2007); see also Ortiz (2010).

⁵ Babbage did not specifically quote Condillac in his works.

1944), in which Condillac's work occupies a central position, the beginning of the nineteenth century is marked by a much closer interest on a systematic development of a theory of signs. This 'general linguistics' was seen as the key to a better understanding, not only of ordinary language, but also of science and of the 'mechanics' of the mind's processes.

The views expressed in de Gérando's four volume book on the theory of signs (de Gérando 1800) mark an important split in method between members of the two successive generations of the French school of *Idéologues*.⁶ This fracture is central to an understanding of the future impact of Babbage's research on functional equations.

The work of de Gérando and, particularly, the far more advanced and refined work of Destutt de Tracy⁷ (to which I will refer briefly in the next section), Maine de Biran and others, then in the same philosophical school, brought about a change of perspective in relation to Condillac's optimistic views on the power of language in science. In addition, I argue that these advances in the perception of language as an analytical method—which no doubt were a French philosophical export of considerable mathematical significance—were not clearly received in Babbage's English mathematical environment until, possibly, some twenty years after they had been formulated in Paris, as suggested by the structure of his work on the calculus of functions of 1815-1816 and by a key paper written by Babbage in 1821, to which I will return later.

Although there are differences, sometimes deep, between the two philosophers, de Gérando's intention was not to demolish Condillac; indeed, much of his work was to remain as an integral part of the school's second generation. This permanence at a time of deep philosophical debate and intense social and political change is one of the elements that give historical coherence to the school of *Idéologie*, a group whose identity and unity are sometimes difficult to grasp.

De Gérando credits Condillac with the perception that the mechanism of abstract thought involves a succession of translations (de Gérando 1800, 1, p. xiv), and also with having started an exploration on the relationship between signs and ideas. However, he criticized Condillac for the excessive emphasis he had placed on the interplay between languages and analysis, a point that may have benefited Babbage's mathematical work if he had known of it earlier.

On that important issue de Gérando was very specific, questioning Condillac's grand ideas on language as an analytical tool quite frontally, rather than referring to possibly correctable limitations or shortcomings in his conceptions. He also censured Condillac for making statements that were too absolute, for example that the study of a science is limited to learning a 'language' and that a well developed science is nothing but a well constructed language.

It should be noted, however, that the perception of the position of language in science was experiencing some change in the twenty years that separate Con-

⁶ I follow Picavet (1891) in the stratification of schools within the *Idéologie* movement.

⁷ On Destutt de Tracy see Arnault *et. al.* (1822), Newton (1852), Picavet (1891), Moravia (1968), Gusdorff (1978), Kennedy (1978).

dillac and his new critical followers. The existence of an intellectual shift can be detected through a careful reading of the views expressed by some scientists of the period, even if they explicitly claim having been inspired by Condillac's works.⁸

De Gérando's work marks the tentative beginnings of a new grouping within Condillac's school. While still accepting some of his basic ideas on sensations, its members began to distance themselves from his attractive, perhaps also simplistic, ideas on the overwhelming power of language to dictate the path of science and knowledge. In particular, they broke with Condillac's implicit view that removing any major conceptual obstacles in a specific field of knowledge may not be a necessary precondition for the construction of a scientific language.⁹

In his book on the theory of signs de Gérando hinted that language alone is not the cause of mathematics' prominence; he suggested that the very special structure of mathematics is what has allowed for the development of a powerful language in it. On systems of language, in particular on algebraic language, de Gérando referred to the coexistence of two very different ingredients: one depending on the nature of the signs used and the other on the laws controlling their composition; in terms of grammar, the elements of the discourse and syntax (de Gérando 1800, 1, p. 263). It became clear to de Gérando that a new methodical language would not be an 'algebra' in the traditional sense of the word, as its structure and rules would not be those of ordinary algebra. For example, methodical languages would not share with the algebraic language the facility of having 'mobile' symbols, as in ordinary algebra, capable of representing with equal validity known and unknown quantities. In a methodical language there cannot be 'unknowns' in the algebraic sense, as the formation of signs is controlled by the formation of the corresponding (and therefore *known*) ideas. He concluded that the symbols and methods of algebra are inapplicable to metaphysics because of the very nature of the topics discussed there (de Gérando 1800, 4, pp. 170-171). De Gérando distanced himself even more clearly from Condillac when he remarked that the symbols and methods of one cannot be easily applied to the other, due to the different nature of the questions addressed in algebra and metaphysics.

5. A further break with Condillac: Destutt de Tracy's conception of languages as special algebras

Destutt de Tracy, from his early works (Destutt de Tracy 1798, 1801), indicated his disagreement with the idea, often derived from Condillac, that to renovate a particular chapter of science and launch it into a new avenue of progress

⁸ Such as in Lavoisier *et. al.* (1787); I will return to this point in the next section.

⁹ In Condillac (1798) the author himself had already encountered difficulties in his discussion of algebra; for example, in connection with the possible enlargement of the realm of quantity with complex numbers, which he was naturally forced to reject.

it is only necessary to renew its nomenclature and introduce a more systematic language, a language *méthodique*. He wrote that “[c]ependant ce n’est point du tout cela dont il s’agit” (Destutt de Tracy 1826b, pp. 26-27). In 1815-1816 Babbage remained one of those distant disciples of Condillac who still upheld the outdated ideas.

Returning to his critique of Condillac’s prescriptions, Destutt de Tracy stated in his *Éléments* (Destutt de Tracy 1801a, b, 1817, 1826a, b) that even accepting the advantages of a good nomenclature and of a well designed language, it is not *words* that *create* science (Destutt de Tracy 1826b, p. 27n). In a direct reference to Lavoisier’s work, he used the theory of the phlogiston to illustrate how little advance had been possible before clarifying the obscure conceptual points of that science to the extent Lavoisier and his colleagues had done. It was only *then* they confronted the task of constructing a new language capable of expressing their new ideas. He had turned Condillac’s propositions upside down. In Babbage’s ‘Essay’, however, basic questions relating to the existence or uniqueness of the inverse of the operator defining a given functional equation are, quite naturally for the time, left aside (Ortiz 2007).

In his work Destutt de Tracy considered some of Condillac’s oversimplifications on the question of languages specifically designed for use in science and discussed the possibility that these languages may have a structure far more complex than that of ordinary algebra. He did not accept that scientific languages (with the exception of that of algebra) could be regarded as potential keys to formulate the way forward in a specific field of science in which its fundamental laws have not yet been fully formulated. In his view scientific languages are representations of a contingent scientific structure. He did not rule out, however, the existence of formal analogies between algebra and the various scientific languages. His view on the relationship between algebra and other specific scientific languages is expressed in the statement: “[c]’est bien là ce qu’est l’algèbre: aussi l’algèbre est une langue, et les langues ne sont elles-mêmes que des espèces d’algèbres” (Destutt de Tracy 1801b, p. 239). Clearly, not *ordinary* algebra as Condillac had postulated. It was left to Georges Boole to construct, in 1847, the special algebra for classical logic (Boole 1847), opening the way to fundamental studies in the field of algebraic logic, which reach to the present.

6. A direct contact: Babbage’s visit to Paris in 1819

According to his biographers,¹⁰ Babbage most probably made his first visit to Paris in 1819, accompanied by his friend John Herschel. In that trip Babbage met with a number of scientists who were in direct contact with leading *Idéologues*, sharing with them positions in many official bodies, including the *Académie des Sciences*.

¹⁰ Hyman (1982), see also Babbage (1864), Chapters 35 and 36.

Many consequences resulted from these contacts. In the same year, through the foundation of the Cambridge Philosophical Society, Babbage and his friends made efforts to root in their country associations with a more modern design.¹¹ After Babbage visited Paris a short extract of his 1815-1816 work was translated into French and published in 1822 by Joseph D. Gergonne (Gergonne 1821-182) in his prestigious mathematical journal; however, it was not followed by further research on the French side.

In a paper Babbage wrote in 1821 (published in 1827) he adopted a more subtle evaluation of the power of language than in his 'Essay' of 1815-1816, becoming closer to the ideas de Gérando developed in his 1800 work on signs. Babbage argued in 1821 (Babbage 1827, pp. 329-330) that he had independently arrived at the views expressed by de Gérando in 1800; this, necessarily, must have happened in the second half of the 1810s, that is, after the writing of his 'Essay' on functional equations.

However, in 1821 his references still show a delay in his intellectual communication with research published in French scientific-philosophical circles. By 1817 de Gérando's critical assessment of Condillac had been largely superseded in France, particularly through the publication of Destutt de Tracy's elegant, well designed and original treatise *Éléments d'Idéologie* (Destutt de Tracy 1801b, 1817, 1826a, b) and, also, through the work of some of his one-time disciples, such as Maine de Biran. By 1821 Babbage does not seem to have been seriously influenced by that more recent work, which suggests again that by that date he still operated with some considerable delay in relation to recent critical analysis on the philosophy of science and mathematics in France.

It is also significant that after the 1820s Babbage stopped working on the development of abstract scientific languages to solve problems in pure mathematics, as he had attempted for the case of functional equations in his original 'Essay' of 1815-1816. In doing that he left an apparently promising line of research that combined pure mathematics with an abstract 'linguistic-algebraic' approach. In fact, by 1820 Babbage abandoned altogether research in pure mathematics.

7. Conclusion

After 1820 Babbage's original methodology of 1815-1816 was not advanced by his work or by work by other mathematicians, and cannot be regarded as having been fully incorporated into the contemporaneous body of mathematical analysis. Perhaps the difficulty of his paper may have been a factor, but clearly it was not the only one. Leaving historians of the mathematics of functional equations aside, Babbage's approach is almost forgotten today. Using a very different and less systematic approach than Babbage's, and clearly unaware of his work, Silberstein solved again, in 1940, one of the types of functional equations explicitly discussed by Babbage in 1815 (Silberstein 1940).

¹¹ Later, Babbage played a leading role in attempts to modernize and reform the Royal Society.

When the so-called ‘symbolic’ techniques began to be developed in Britain, from the 1830s onwards,¹² there was little reference to Babbage’s pioneering work on functional equations.¹³ Perhaps, views on language, mainly through references to the works of Destutt de Tracy and his French and foreign followers, had changed sufficiently to make Babbage’s original theoretical framework look unappealing.

However, if not strictly in pure mathematics, through his scientific life Babbage continued using different forms of a general concept of ‘language’, often expressed in terms of the notion of algorithm (Grattan-Guinness 1992), and mainly as a research tool for his highly innovative work on the automation of mechanical calculations. He developed a specific language to facilitate the reading of his drawings for the construction of machine parts for his imaginative calculating machinery and, later, his associate Ada Lovelace attempted to develop a machine-programming language.

There is a further point of historical interest in relation to Babbage’s work on functional equations in the decade of 1810 to 1820. It is well known that Babbage belongs to the minority of English mathematicians generally regarded as acquainted with—or at least interested in—the new developments that were taking place contemporaneously in France. As the period considered in this paper covers from the last years of the Napoleonic era to the early years of the Bourbon Restoration, through Babbage’s case we can derive some direct historical information on the impact of international conflicts on channels of scientific communication between France and England.

Considering the history of scientific relations between these two countries in that precise period some authors have emphasized the view that ‘science was never at war’¹⁴. In view of what has been said in this paper it seems reasonable to question the real strength of the intellectual channels of communication between these two countries in that important period. Clearly, in this questioning I limit my remarks to a very narrow field: exchanges in areas of mathematics and philosophy that affected pure mathematics research.

¹² Particularly through the work of George Peacock, Augustus De Morgan, and Duncan F. Gregory in the 1830s; see Nový (1973), Ch. 6; see also Ruffieux (2005).

¹³ Grattan-Guinness (1979), p. 86, in his detailed review of Dubbley (1978), points to a short reference of Babbage’s work in De Morgan (1835?).

¹⁴ Such as in the influential work of de Beer (1960); for a comprehensive analysis of the relations between these two countries in the late eighteenth century see Crosland (2005).

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